

Reduction of the Signal Nonlinear Distortion in CATV Systems applying Dual Mach-Zehnder Modulators

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Summary:

The paper deals with some investigations aimed at applying compensation techniques to reduce nonlinear signal distortion in the Mach-Zehnder modulator of HFC CATV systems. The frequency distribution of second- and third-order intermodulation products of N-channel transmission systems is shown. Models of dual-parallel and dual-cascade Mach-Zehnder modulators are suggested based on both the mathematical model of a conventional Mach-Zehnder modulator and the features of the CATV system itself, thus making it possible for the modulator parameters to be determined in a way to ensure CSO products cancellation and CTB products minimization. The dependence of the C/CTB parameter on the modulation index is studied for N-channel HFC CATV systems applying a dual-parallel or dual-cascade Mach-Zehnder modulator. Optical channels implementing dual-parallel or dual-cascade Mach-Zehnder modulator instead of a conventional one are compared and the possibilities to improve the RF signals' dynamic range are analyzed. Relations to determine the increase of the optical loss introduced by dual Mach-Zehnder modulators are suggested that help compare it with that caused by a conventional Mach-Zehnder modulator.

1 Introduction

Analog transmission of RF signals over the optical channel of a hybrid fiber coaxial (HFC) CATV system can be based on either direct laser modulation or an external modulator. The parameters of the optical channels with direct laser modulation are of poor quality due to laser chirping, nonlinearity and slightly sloping transfer characteristic etc. To eliminate such a disadvantage a laser with a constant bias current and an external modulator at its output is used.

Modern HFC CATV systems usually apply electro-optical intensity modulators based on the Mach-Zehnder interferometer and known as MZ-modulators (MZM). The linear part of the conventional MZM transfer characteristic is rather short which results in nonlinear distortion of the signals transmitted over the optical channel. With HFC CATV systems the carrier-to-composite second order (C/CSO) products and the carrier-to-composite triple beat (C/CTB) products ratio measured at the optical channel output is required to be higher than 60 dB. To provide the minimum CSO and CTB values needed the conventional MZM must be operated with a modulating RF signal of

comparatively small amplitude which however causes the carrier-to-noise ratio (CNR) to decrease, hence the received information quality to worsen.

Different methods for linearization of the MZM transfer characteristic have been developed. They improve the dynamic range of the input RF signals and keep the carrier-to-intermodulation distortion ratio within the required limits. Linearization techniques most often apply several conventional MZMs (two as usual) whose mode of operation is set in a way to achieve an effective suppression of the nonlinear distortion products [1–3]. The method usually applied to determine the parameters of such modulators consists in representing their transfer characteristic as a power series. Such an approach however neglects the peculiarities of the systems here considered. Because of that in the work a mathematical model based on Bessel functions has been used to describe the MZM [4]. The purpose was to optimize the modulator parameters in a way to reduce the nonlinearities over a broad frequency band and to maximize the modulation efficiency, i.e. to maximize the linearity of its transfer characteristic and to minimize the optical loss.

2 Frequency distribution of nonlinear distortion products

Analysis has shown that CSO and CTB products at frequencies $\omega_i + \omega_j$, $\omega_i - \omega_j$, $\omega_i + \omega_j - \omega_k$, $\omega_i - \omega_j + \omega_k$ and $\omega_i - \omega_j - \omega_k$ ($\omega_i < \omega_j < \omega_k$) must be taken into consideration when determining the nonlinear distortion whereas products at frequencies $2\omega_i$, $3\omega_i$, $2\omega_i \pm \omega_j$, and $\omega_i + \omega_j + \omega_k$ turn out to be of no interest because they fall out of the frequency band or are of a negligible level [5].

The number of CSO products to appear in a RF channel can be determined as follows:

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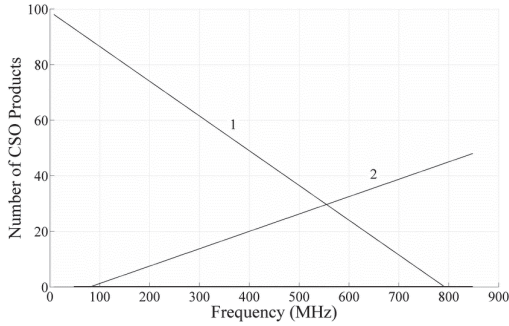


Fig. 1: Distribution of CSO products:
1 – CSO($\omega_i - \omega_j$) and 2 – CSO($\omega_i + \omega_j$)

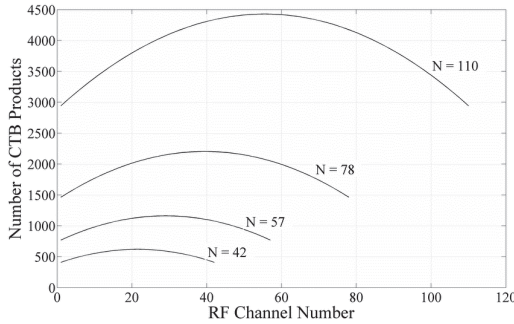


Fig. 2: Number of the CTB products to appear in one RF channel

$$\begin{aligned} N_{CSO(\omega_i - \omega_j)} &= (N - 1) [1 - (f - \Delta f)/(f_H - f_L)] \\ N_{CSO(\omega_i + \omega_j)} &= 0.5 (N - 1) (f - 2f_L + \Delta f)/(f_H - f_L) \end{aligned} \quad (1)$$

where N is the carriers number, f is the frequency of the examined channel, f_L and f_H are the frequencies of the lowest and the highest frequency channel respectively and Δf is the carrier spacing.

In Fig. 1 the frequency distribution of CSO products is shown, where $f_L = 111$ MHz, $f_H = 855$ MHz, $\Delta f = 8$ MHz and $N = [(f_H - f_L)/\Delta f + 1] = 94$. It is obvious that the number of CSO products is at its maximum for the lowest and the highest frequency channel, its rise being twice as big for the lowest channels. It is important to note that when the number of RF channels is reduced, the number of intermodulation products is reduced too.

To calculate the number of third-order intermodulation distortion products at frequencies $\omega_i \pm \omega_j \pm \omega_k$ the following expression can be used:

$$N_{CTB} = 0.25(N - 1)^2 + 0.5(N - M)(M - 1) - 0.25N \quad (2)$$

where M is the number of the RF channel. The frequency distribution of a broadband system for a different number of channels transmitted is shown in Fig. 2.

By solving equation $dN_{CTB}(M)/dM = 0$ one can show that a maximum number of the CTB products is attained when $M = (N + 1)/2$, hence the CTB products number is at its maximum for the central RF channel and can be calculated with the following formula

$$N_{CTB(\max)} = (3N^2 - 8N + 2)/8. \quad (3)$$

If $N \gg 1$, then equation (3) can be written in the form $N_{CTB(\max)} \approx 3N^2/8$.

3 Nonlinear signal distortion in conventional MZM

The MZM transfer characteristic is described by the following equation:

$$P_{out} = 0,5 P_{in} \left[1 + \cos \left(\frac{\pi u_{mod}(t)}{U_\pi} - \varphi_{bias} \right) \right] \quad (4)$$

where P_{in} is the input optical power, $u_{mod}(t)$ is the modulator driving voltage, φ_{bias} is the static phase shift (bias point) and U_π is the half-wave switching voltage which could be calculated as

$$U_\pi = \frac{\lambda}{2 n^3 r l} \quad (5)$$

where λ is the optical wavelength in vacuum, n is the index of refraction, r is the electro-optic coefficient of the material (LiNbO₃, LiTaO₃, etc.), l is the electrode length and d is the electrode gap.

The static phase shift depends on both the bias voltage (U_{bias}) and the internal pathlength mismatch of the two arms of the interferometer $\delta = n_1 l_1 - n_2 l_2$. To calculate φ_{bias} the following formula can be used:

$$\varphi_{bias} = \pi U_{bias}/U_\pi + 2\pi \delta/\lambda. \quad (6)$$

At best i.e. when $\delta \rightarrow 0$ the modulator operates in the quadrature bias point ($\varphi_{bias} = -\pi/2$) and $U_{bias} = -U_\pi/2$. The required bias voltage U_{bias} can be easily calculated if (5) is used.

When N carriers of equal amplitude U_{mod} are transmitted over the optical channel and the modulator is assumed to operate in quadrature, (4) can be expressed as:

$$\begin{aligned} P_{out} &= 0,5 P_{in} \left[1 + \cos \left[m \sum_{i=1}^N \sin(\omega_i t) \right] \cos(\varphi_{bias}) + \right. \\ &\quad \left. + \sin \left[m \sum_{i=1}^N \sin(\omega_i t) \right] \sin(\varphi_{bias}) \right] \end{aligned} \quad (7)$$

where $m = \pi U_{mod}/U_\pi$ is the modulation index.

The levels of the spectral terms of the output optical power can be determined if formula (7) is expanded in a power series [6] as follows:

$$\begin{aligned} \sin(x \sin \Theta) &= 2 \sum_{n=1}^{\infty} J_{2n-1}(x) \sin[(2n-1)\Theta] \\ \cos(x \sin \Theta) &= J_0(x) + 2 \sum_{n=1}^{\infty} J_{2n}(x) \cos 2n\Theta \end{aligned} \quad (8)$$

where $J_n(x)$ is the Bessel function of the first kind and of order n of the variable x . Then the power (first harmonic) of the output signal of a frequency ω_i will be

$$P_{\omega_i} = P_{in} J_1(m) J_0^{N-1}(m) \sin(\varphi_{bias}). \quad (9)$$

To calculate the power of the CSO terms at frequencies $\omega_i \pm \omega_j$ and of the CTB terms at frequencies $\omega_i + \omega_j - \omega_k$ and $\omega_i - \omega_j \pm \omega_k$ the following expressions are used:

$$\begin{aligned} P_{\omega_i \pm \omega_j} &= P_{in} J_1^2(m) J_0^{N-2}(m) \cos(\varphi_{bias}) \\ P_{\omega_i \pm \omega_j \pm \omega_k} &= P_{in} J_1^3(m) J_0^{N-3}(m) \sin(\varphi_{bias}) \end{aligned} \quad (10)$$

where i, j , and k are integer and correspond to the number of the RF channels transmitted.

It is obvious that when $U_{bias} = \pm 0.5U\pi$ ($\varphi_{bias} = \pm \pi/2$) no CSO product appears in the conventional MZM. Hence it is enough to calculate the C/CTB ratio in the central RF channel in order to estimate nonlinear distortion.

The phases of the CTB products in each RF channel being a random value the root mean square (rms) values must be used to calculate the total CTB products power. Therefore the C/CTB ratio for the central RF channel in a broadband HFC CATV system can be calculated as

$$C/CTB = [J_0(m)/J_1(m)]^4 (N_{CTB \max})^{-1} \quad (11)$$

where $N_{CTB \max}$ is defined by (3).

Formula (11) makes it possible for the modulation index m to be determined if known the number of the transmitted channels N and the minimum permissible value of the C/CTB ratio ($C/CTB \geq 60$ dB for the systems here considered). The results show that the maximum acceptable value of m is about 0.01. Such a low modulation index however degrades the signal-to-noise ratio at the optic receiver input i. e. it causes the information received to worsen. Hence, more sophisticated MZM circuits are indispensable to lengthen the linear part of the modulation characteristic.

4 Parameters of the dual parallel MZM to provide nonlinear distortion compensation

The dual parallel MZ-modulator (DPMZM) shown in Fig. 3 consists of a primary modulator (MZM1) and a compensative sub-modulator (MZM2) that are connected in parallel optically and electrically. The optical input power splits between MZM1 and MZM2 in ratio $s/(1-s)$ the aim being to make the value of the optical power splitting ratio s be as close to 1 as possible, thus minimizing the optical power loss. The compensative sub-modulator electrodes are k times longer than those of the MZM1, i. e. the amplitude of the modulating signal is k times bigger which makes the signal distortion to increase. Setting an appropriate value to bias voltages U_{bias1} and U_{bias2} will make both modulators operate in quadrature, i. e. $\varphi_{bias1} = -\pi/2$ and $\varphi_{bias2} = \pi/2$. Thus the phase of the MZM2 output signal will shift π radians in relation to

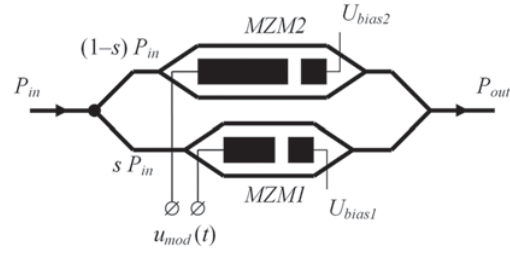


Fig. 3: Dual parallel Mach-Zehnder modulator

MZM1 and the nonlinear distortion products in MZM2 will compensate for those in MZM1.

On the basis of the above considerations and (7) the DPMZM transfer characteristic can be rewritten as follows:

$$\begin{aligned} P_{out} &= s \frac{P_{in}}{2} \left\{ 1 + \sin \left[m \sum_{i=1}^N \sin(\omega_i t) \right] \right\} \\ &\quad - (1-s) \frac{P_{in}}{2} \left\{ 1 + \sin \left[km \sum_{i=1}^N \sin(\omega_i t) \right] \right\}, \end{aligned} \quad (12)$$

where k is the electrode length ratio of MZM2 and MZM1. The output power terms can be determined by developing the last equation into a power series by means of (8).

Since both modulators MZM1 and MZM2 work in quadrature, they do not generate CSO products. Hence here again the carrier-to-CTB products power ratio can be used to evaluate the modulator nonlinearity which is most significantly influenced by the CTB products at frequencies $\omega_i + \omega_j - \omega_k$ and $\omega_i - \omega_j \pm \omega_k$.

On the basis of the relations obtained for the fundamental output signal (P_{ω_i}) and the rms power of the CTB products ($P_{\omega_i \pm \omega_j \pm \omega_k}$) the following formula for the C/CTB ratio in the central RF channel is developed:

$$\begin{aligned} \frac{C}{CTB} &= \\ &= \frac{\left[s J_1(m) J_0(m)^{N-1} - (1-s) J_1(km) J_0(km)^{N-1} \right]^2 3N^2}{\left[s J_1(m)^3 J_0(m)^{N-3} - (1-s) J_1(km)^3 J_0(km)^{N-3} \right] 8} \end{aligned} \quad (13)$$

Parameters s and k can easily be optimized using (13) so that the C/CTB ratio be kept bigger than a given permissible minimum value over the whole operating radio-frequency range of the HFC CATV system. To do this one should proceed as follows:

First, the variation limits of the modulation index m must be determined. It is well known that increasing m improves the CNR, yet it does increase impairment caused by nonlinearity too. Hence, the optimum operating value of m is a balance between noise and distortion. With the systems here considered a rather short variation interval (0.03 to 0.06) of the modulation index m provides for the admissible minimum value of the CNR and C/CTB parameters [7].

Table 1: The splitting ratio values versus k

k	1.5	2.0	2.5	3.0	3.5
s_{\max}	0.77	0.88	0.93	0.96	0.97
s_{\min}	0.76	0.87	0.92	0.94	0.95

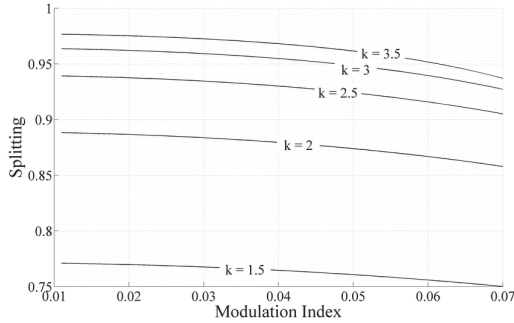


Fig. 4: Optical power splitting ratio versus modulation index under k

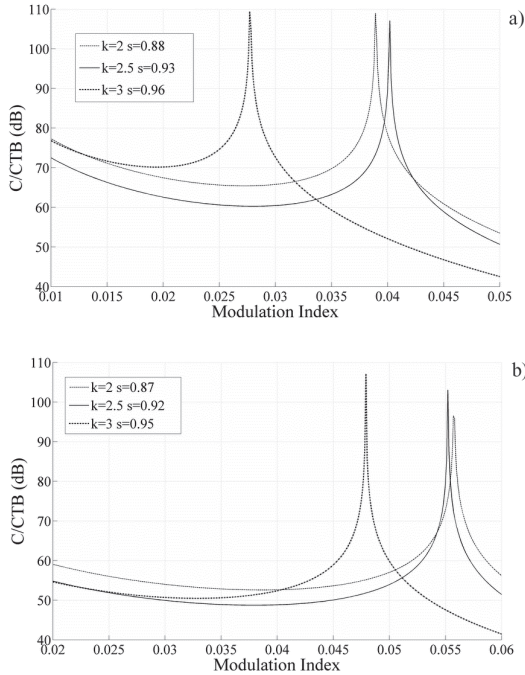


Fig. 5: a) and b) Determination of the optimum value of parameters k and s

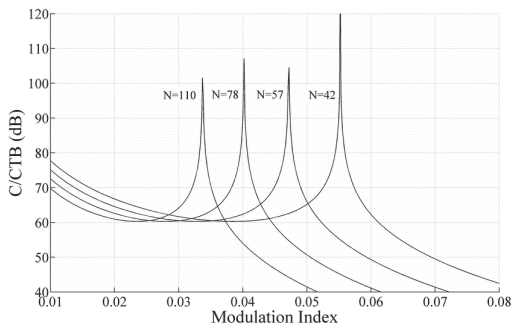


Fig. 6: Dependences of the DPMZM C/CTB ratio of on the modulation index and the number of transmitted RF channels for $k = 2.5$ and $s = 0.93$

If DPMZM is assumed to be an ideal one, i.e. a total compensation of all CTB products is achieved, the dependence of the optimum of the optical power splitting ratio s on parameter k and the modulation index m can be found. In that case the denominator of (13) is equal to zero, hence the following formula holds:

$$s = \frac{J_1(km)^3 J_0(km)^{N-3}}{J_1(m)^3 J_0(m)^{N-3} + J_1(km)^3 J_0(km)^{N-3}} \quad (14)$$

In Fig. 4 the optical power splitting ratio s is shown as a function of the modulation index m for five values of k . It is obvious that the bigger the electrode length ratio k the smaller the optical power loss in the modulator. Besides, for a fixed value of k (see Table 1) parameter s is slightly changing within the admissible limits of m (0.03 to 0.06).

To determine the optimum values of s and k the dependence of the C/CTB parameter on the modulation index m for a different number N of the transmitted RF channels has been investigated. In Fig. 5 the obtained results are shown for $N = 78$. The values of parameters k and s are chosen according to the above considerations, Fig. 5a referring to $s = s_{\max}$ and Fig. 5b referring to $s = s_{\min}$ respectively.

As seen from the diagrams, the requirements for the maximum value of m and the minimum value of the optical loss in DPMZM are met simultaneously when $k = 2.5$ and $s = 0.93$. Then $C/CTB \geq 60$ dB if $m \leq 0.044$, whereas if bigger values of k and s ($k = 3.0$ and $s = 0.95$) are chosen to minimize the optical loss then the maximum permissible value of m is 0.035. Varying the number of the transmitted RF channels produces no effect on the investigated parameters' optimum – the values are still $k = 2.5$ and $s = 0.93$.

The following expression can be used to evaluate the increase of the optical loss in DPMZM in respect to the conventional MZM:

$$P_{\omega_i} \approx P_{\omega_i}^{conv} [s - (1 - s)k]. \quad (15)$$

It follows that when $k = 2.5$ and $s = 0.93$ the additional loss in the DPMZM is about 1.2 dB.

Nonlinear distortion of the DPMZM signals for $k = 2.5$, $s = 0.93$ and $N = 42, 57, 78, 110$ can be estimated from Fig. 6. The admissible maximum values of m for which condition $C/CTB \geq 60$ dB holds are given on Table 2. On the same table the values of m corresponding to a conventional MZM and obtained with (11) are shown. Thus the possibilities to linearize the modulation characteristic of the DPMZM here considered can be analyzed. It

Table 2: The admissible maximum values of m for which condition $C/CTB \geq 60$ dB holds

m_{\max} for $C/CTB = 60$ dB				
N	110	78	57	42
MZM	0.007	0.008	0.010	0.011
DPMZM	0.037	0.045	0.052	0.060
DCMZM	0.042	0.050	0.059	0.068

is obvious that if a DPMZM is used the level of the input RF drive voltage can be about 5 times higher than that with a conventional MZM.

5 Possibilities to reduce nonlinear distortion by applying dual cascade MZM

There are different ways to cascade two conventional MZMs in order to form a compensation device, known as dual cascade Mach-Zehnder modulator (DCMZM) [8]. The paper deals with the type of DCMZM for which investigations carried out on the transfer characteristic linearization turn out to be most satisfactory in comparison with others.

The cascade modulator here discussed is shown in Fig. 7. It can be characterized as follows: both conventional MZ modulators do not operate in quadrature bias point; the output optical power of the first-stage sub-modulator MZM1 does not split equally between the arms of the second-stage sub-modulator MZM2; an antiphase split is used to load the two stages of the DCMZM.

Taking into account the above peculiarities the output power can be expressed as

$$P_{out} = \frac{P_{in}}{4} \left\{ 1 + \cos \left[m \sum_{i=1}^N \sin(\omega_i t) + \varphi_{bias1} \right] \right\} \cdot \left\{ 1 + a \cos \left[-m \sum_{i=1}^N \sin(\omega_i t) + \varphi_{bias2} \right] \right\} \quad (16)$$

where a is the cross-over coefficient of the directional coupler (DC).

To determine the fundamental signal power at frequency ω_i and the terms of a second ($P_{\omega_i \pm \omega_j}$) and third ($P_{\omega_i \pm \omega_j \pm \omega_k}$) order a trigonometric transformation and the Bessel function expansion (8) must be applied to equation (16). The result is as follows:

$$P_{\omega_i} = \frac{P_{in}}{2} \left[(Q - aS) J_1(m) J_0^{N-1}(m) - 0.5a(QR - PS) J_1(2m) J_0^{N-1}(2m) \right] \quad (17)$$

$$P_{\omega_i \pm \omega_j} = \frac{P_{in}}{2} \left[(P + aR) J_1^2(m) J_0^{N-2}(m) + 0.5a(PR + QS) J_1^2(2m) J_0^{N-2}(2m) \right] \quad (18)$$

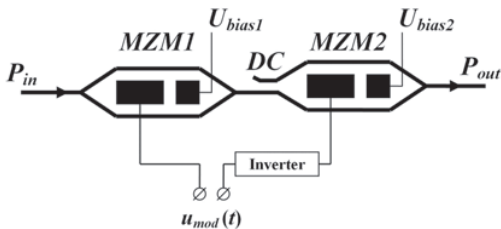


Fig. 7: Dual cascade Mach-Zehnder modulator

$$P_{\omega_i \pm \omega_j \pm \omega_k} = \frac{P_{in}}{2} \left[(Q - aS) J_1^3(m) J_0^{N-3}(m) - 0.5a(QR - PS) J_1^3(2m) J_0^{N-3}(2m) \right] \quad (19)$$

where $P = \cos(\varphi_{bias1})$, $Q = \sin(\varphi_{bias1})$, $R = \cos(\varphi_{bias2})$, and $S = \sin(\varphi_{bias2})$.

To find the optimum values of parameters a , φ_{bias1} , and φ_{bias2} when linearizing the DCMZM transfer characteristic several steps must be taken. First, after nullifying the coefficients before the Bessel function in (18) the following relations for the examined parameters are obtained:

$$\begin{cases} P + aR = \cos(\varphi_{bias1}) + a \cos(\varphi_{bias2}) = 0 \\ PR + QS = \cos(\varphi_{bias1}) \cos(\varphi_{bias2}) + \sin(\varphi_{bias1}) \sin(\varphi_{bias2}) = 0 \end{cases} \quad (20)$$

If the second equation is written in the form $\cos(\varphi_{bias1} - \varphi_{bias2}) = 0$ then a relation $\varphi_{bias2} = \varphi_{bias1} - \pi/2 + k\pi$ will follow. When the latter is replaced in the first equation the following conditions for the CSO products' cancellation are derived:

$$\begin{cases} \varphi_{bias2} = \varphi_{bias1} - \pi/2 \\ a = -\cotg(\varphi_{bias1}) \end{cases} \quad (21)$$

Furthermore, (21) are applied to (19) and the power of the CTB term is obtained in the following reduced form:

$$P_{\omega_i \pm \omega_j \pm \omega_k} = -\frac{P_{in}}{2} \left[\left(Q - \frac{P^2}{Q} \right) J_1^3(m) J_0^{N-3}(m) - 0.5 \frac{P}{Q} J_1^3(2m) J_0^{N-3}(2m) \right] \quad (22)$$

Then (22) is used to determine the values of parameter φ_{bias1} for which, given the number N of RF channels transferred and the value of m , function (22) is at its minimum. The result obtained is $\varphi_{bias1} = -0.421\pi$. The values of the other two parameters obtained with formula (21) are $\varphi_{bias2} = -0.921\pi$ and $a = 0.254$.

Figure 8 illustrates how the C/CTB products' power ratio of the DCMZM under consideration will change as a function of the modulation index and the number of RF channels. The admissible maximum values of m for which condition $C/CTB \geq 60$ dB holds for that type of modulator are shown on Table 2 too. Hence, the DCMZM under consideration enables the amplitude of the drive voltage to increase about 6 times if compared to the conventional modulator.

Though DCMZM provides a longer linear slope of the modulation characteristic its optical loss turns out to be greater than that of the DPMZM. The increase of signal power loss in a DCMZM if compared to the conventional MZM can be evaluated as follows:

$$P_{\omega_i} \approx P_{\omega_i}^{conv} \left[\frac{P^2 + P - Q^2}{2Q} \right] \quad (23)$$

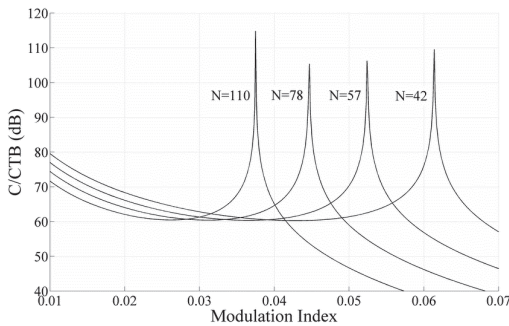


Fig. 8: Dependences of the DCMZM C/CTB ratio on m and N for $\varphi_{bias1} = -0.421\pi$, $\varphi_{bias2} = -0.921\pi$ and $a = 0.254$

Inserting the values of the chosen DCMZM parameters in (23) will show that the additional loss is about 4.4 dB which makes about 3 dB more than that of the DPMZM.

6 Conclusion

In order to optimize the parameters of the investigated dual-parallel and dual-cascade MZ modulators conditions for the CSO products cancellation and for the minimization of the CTB products power have been used which account for the modulator operating mode, the modulation index and the number of RF channels transmitted.

The investigations have shown that the CSO products cancellation occurs in the DPMZM if both modulators work in quadrature, i. e. $\varphi_{bias1} = -0.5\pi$, $\varphi_{bias2} = +0.5\pi$ whereas in the case of DCMZM this is possible if conditions (21) are fulfilled. The best results for the modulation characteristic linearity and the minimum loss

of the optical power in DPMZM can be obtained when the optical power splitting ratio s and the electrode length ratio k are as follows: $k = 2.5$ and $s = 0.93$. With DCMZM the maximum dynamic range of the input RF signals is achieved if the modulator parameters are as follows: $\varphi_{bias1} = -0.421\pi$, $\varphi_{bias2} = -0.921\pi$ and $a = 0.254$.

The techniques here considered enable the optical channel of the CATV systems to obtain rather satisfactory parameters, the modulation index values being about 5 or 6 times (for DPMZM and DCMZM respectively) better than those with a conventional MZM. Regardless of the increased loss caused by DCMZM these modulators are of great interest for modern communications and must be subjected to further investigation.

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